ESTIMATION OF SIGNAL CORRELATION IN ANTENNA ARRAYS

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Abstract

The correlation between signals received with different antenna elements is an important parameter in an adaptive array. In this paper it is shown that when the signal DOA distribution is uniform the signal correlation can be calculated using measured S-parameters of the antenna input ports. When the spread of the angle of arrival is known, the scattering parameters allow also to estimate the correlation level.

Introduction

The correlation between received signals is a basic parameter for the use of an antenna array or an antenna pair in diversity systems. In many systems the signal can arrive from different directions. In the case of uniform distribution of the angle of arrival the correlation matrix between element port responses can be simply calculated from the scattering matrix measured by a vector network analyzer. The result is equal to that obtained from array element pattern measurements. This is shown by a formula based on the law of energy conservation. The correlation matrix can be found by rescaling the radiated power matrix. The correlation calculated in this manner, the pattern correlation, defines the estimation for the minimum correlation level that can be reached with an antenna pair or with a dual-polarized antenna. Naturally this level decreases with increasing element spacing and increases with narrowing signal distribution.

We show with microstrip array prototypes that the correlation matrices calculated using scattering matrices or element patterns measured in the horizontal plane agree well. This indicates that for the correlation calculations in the case of many sources at the horizontal plane the scattering matrix is useful. Further we demonstrate, how the equality of the correlations calculated using scattering matrix or element patterns allows one to estimate the signal envelope correlation dependency on the absolute value $|S_{12}|$ of the scattering parameter between an antenna pair. The mutual coupling coefficient $|S_{12}|$ is very easy to measure and gives thus a simple method to estimate the signal correlation between the antenna elements in an antenna pair. The envelope correlation level of about 0.7 is usually defined as a practical limit to obtain significant diversity gain with signal combining [1]. For this correlation limit we have estimated the dependency of the allowed levels of the scattering parameter $|S_{12}|$ on the angular spread of the received signal.

Theory

Using the energy conservation law and its independence on array input coefficients we can write for a nondissipative antenna system a matrix formula between the scattering matrix **S** and the matrix of element patterns **F** (see [2])

$$\mathbf{I} - \mathbf{S}^H\mathbf{S} = \mathbf{F}^H\mathbf{F} \tag{1}$$

The element patterns in **F** are vectors defined as scattering parameters of discrete antenna ports of referent antenna in different far field positions and they should be perfectly calibrated. The correlation (coherence) between two pattern vectors $\mathbf{f}_i$ and $\mathbf{f}_j$ can be defined

$$r_{ij} = \frac{\mathbf{f}_i^H\mathbf{f}_j}{\sqrt{\mathbf{f}_i^H\mathbf{f}_i} \cdot \sqrt{\mathbf{f}_j^H\mathbf{f}_j}}. \tag{2}$$

To have the pattern correlation matrix the diagonals in matrices $\mathbf{I} - \mathbf{S}^H\mathbf{S}$ and $\mathbf{F}^H\mathbf{F}$ should be scaled to unity. This can be done by multiplying each column and row with a scaling factor of the corresponding element. Using the scaling we can write the pattern correlation matrix $\mathbf{R}_{\text{pat}}$ based on scattering parameters or on element patterns

$$\mathbf{R}_{pat} = \mathbf{D}^{-1}\left(\mathbf{I} - \mathbf{S}^H\mathbf{S}\right)\mathbf{D}^{-1} = \mathbf{D}^{-1}\mathbf{F}^H\mathbf{F}\mathbf{D}^{-1} = \mathbf{F}_0^H\mathbf{F}\,. \tag{3}$$

Here the scaling matrix **D** is a diagonal matrix containing the square roots of the diagonals of matrix $\mathbf{I} - \mathbf{S}^H\mathbf{S}$ or $\mathbf{F}^H\mathbf{F}$. Usually in theoretical calculations reflected power is not considered and the pattern correlation matrix $\mathbf{R}_{\text{pat}}$ is defined as $\mathbf{F}_0^H\mathbf{F}_0$, where the pattern matrix $\mathbf{F}_0$ contains pattern vectors $\mathbf{f}_{0i}$, pre-scaled to unity norm. However, in the case of pattern pre-scaling the connection to scattering parameters is usually lost. In the case of a 2-element array we can write by Eq. (3) the correlation between patterns of antenna elements 1 and 2

$$r_{12} = -\frac{S_{11}^* S_{12} + S_{12}^* S_{22}}{\sqrt{1 - |S_{11}|^2 - |S_{12}|^2} \cdot \sqrt{1 - |S_{22}|^2 - |S_{12}|^2}}\,. \tag{4}$$

Here we use the fact, that in reciprocal systems $S_{12} = S_{21}$. It is simple to see, that the coefficients r_{12} and r_{21} are complex conjugates and become real in the case of antenna pair symmetry. This formula is convenient to use and gives exact results. With close spacing the mutual coupling distorts the patterns and integration of theoretical model patterns can give poor approximation. Additionally, the complete measurement of patterns with all directions and two polarizations is tedious compared to scattering parameter measurements.

Using absolute values and relative phases of the scattering parameters we can also find another presentation for r_{12}

$$|r_{12}|^2 = \frac{|S_{12}|^2 \cdot |S_{11}|^2 \left[1 + k^2 + 2k\cos(\varphi_1 + \varphi_2)\right]}{\left(1 - |S_{11}|^2 - |S_{12}|^2\right) \cdot \left(1 - |S_{22}|^2 - |S_{12}|^2\right)}\,, \tag{5}$$

where $S_{11}^* S_{12} = |S_{11}| \cdot |S_{12}| e^{j\varphi_1}$, $S_{22}^* S_{12} = |S_{12}| \cdot |S_{22}| e^{j\varphi_2}$ and $|S_{22}| = k|S_{11}|$. We see, that the correlation is always zero, when some of the scattering parameters is zero. Also we find, that it can be zero with special relative phases φ_1 and φ_2, and with special ratio k between S_{11} and S_{12}. To find k and phases φ_1 and φ_2 for minimum correlation, we can write $1 + k^2 + 2k\cos(\varphi_1 + \varphi_2) = 0 \;\Rightarrow\; \cos(\varphi_1 + \varphi_2) = -\frac{1 + k^2}{2k}$. For any $k \geq 0$ this function is always negative and it has a maximum value –1 at $k = 1$, which means that in optimal case the self-matching parameters are the same, $|S_{11}| = |S_{22}|$, and $\varphi_1 + \varphi_2 = 180°$ (see [3]). For a symmetric antenna pair $S_{11} = S_{22}$ and the correlation is zero, when $\varphi_1 = \varphi_2 = 90°$. For the symmetric case we can write

$$r_{12} = -\frac{2\,\mathrm{Re}\{S_{11}^* S_{12}\}}{1 - |S_{11}|^2 - |S_{12}|^2} = \frac{2|S_{11}||S_{12}|\cos\varphi_1}{1 - |S_{11}|^2 - |S_{12}|^2} = \mathbf{f}_{01}^H \mathbf{f}_{02}\,, \tag{6}$$

where $\|\mathbf{f}_{01}\| = \|\mathbf{f}_{02}\| = 1$ and we see, that the correlation can have only real values in the case of symmetry. Here we see also, that if we keep the patterns unchanged, the elements can not be matched, and, if we match the elements, the patterns should change, become orthogonal to each other. With increasing coupling the independence of pattern shapes on matching circuits of antenna elements becomes more and more unrealistic. For a resonant type antenna the parameters S_{11} and S_{22} depend strongly on the frequency. In such case we can approximate their absolute values of S_{11} and S_{22} with some constant, as for example, $|S_{11}| = 0.1$ or $|S_{11}| = 0.5$ to estimate the maximum allowed value of $|S_{11}|$ for a given correlation level.

The phase angle of S_{12} depends on the distance between the elements and thus the correlation oscillates with increasing antenna element separation. We can, however, assume some characteristic cases of $\cos\varphi$ (like $\cos\varphi = 1$, which removes the oscillations of correlation), and find the “safe” dependency of r_{12} on $|S_{12}|$.

The envelope correlation ρ_{env} is about the same as the square of the complex correlation r_{12} ([1], [4])

$$\rho_{env} \approx |r_{12}|^2 . \tag{7}$$

Results

In Fig. 1 and 2 we see experimental results for correlation coefficients between different elements in linear microstrip arrays of 6 elements depending on element distance d_{ij}. The element spacing is from 0.3λ to 0.9λ in these arrays. The correlation coefficients r_{ij} are from the matrices in Eq. (3). In Fig. 1 the scattering matrix (left) part is used and in Fig. 2 the pattern (right) part of Eq. (3) is used.

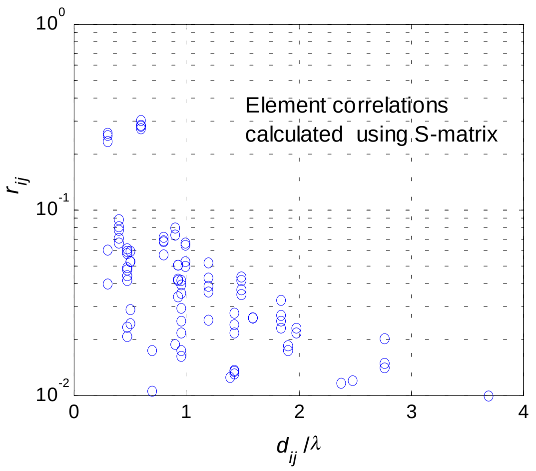

Fig.1 Distance dependency of array element correlation for all elements in different arrays calculated using scattering matrix

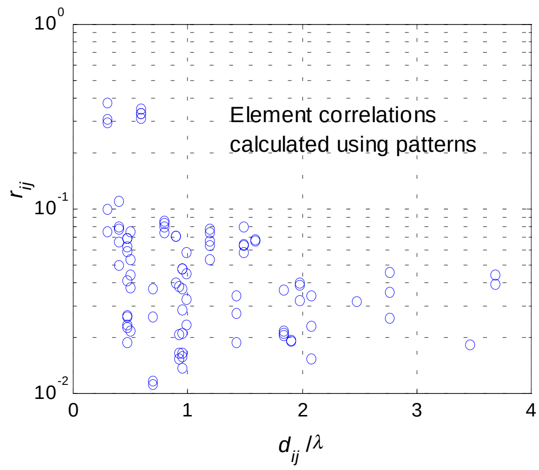

Fig.2 Distance dependency of array element correlation for all elements in different arrays calculated using measured patterns

The patterns are measured in the horizontal plane and we see, that in this approximation the results agree well. With lower correlation values the results using measured patterns are somewhat random compared with results using the scattering matrix. However, we can conclude, that the horizontal plane approximation of patterns can be used for microstrip arrays, which is of practical interest (see [1]).

The safe estimation using $\cos\varphi = 1$ in (6) gives the dependency of the envelope correlation on $|S_{12}|$ in the case of uniform distribution of angle of arrival. It is presented in Fig. 3 for different levels of $|S_{12}|$. Further the exact envelope correlation is calculated with pattern integration using ideal uniform planar patterns for different element distances and different angular spread of the angle of arrival. For each element distance the angular spread, which gives correlation $\rho_{env} = 0.7$ is sought. Then the corresponding level of the envelope correlation for 180° angular spread for the same element distance is calculated. For this envelope correlation the corresponding $|S_{12}|$ is calculated by (6) with $|S_{11}| = 0.5$ and $\cos\varphi = 1$. The resulting dependency of $|S_{12}|$ on the angular spread θ_{spread} is the oscillating curve presented in Fig. 4. Because the correlation is an oscillating function as $\cos\varphi$, the safe level of $|S_{12}|$ to guarantee the fixed correlation level would be a smooth curve across the maximums. Also another way to estimate the dependency $\rho_{env}(|S_{12}|)$ is used. The envelope correlation is assumed to grow linearly to critical value 0.7 with decreasing angular spread θ_{spread} by formula

$$\rho_{env}(\theta_{spread}) = \frac{180^\circ}{\theta_{spread}} \rho_{env}(\theta_{spread} = 180^\circ) . \tag{8}$$

With this formula the envelope correlation in the case of uniform distribution of arrival angle $\rho_{env}(\theta_{spread}=180°)$ is calculated with $\rho_{env}(\theta_{spread}) = 0.7$. The calculated level of envelope correlation with $\theta_{spread} = 180°$ is connected to the level of $|S_{12}|$ using Eqs. (6) and (7). The result is the smooth curve in Fig.4.

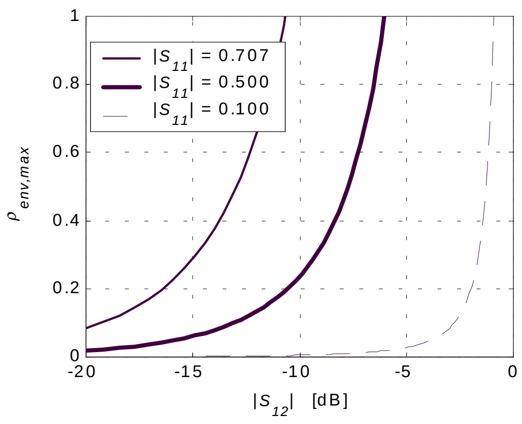

Fig. 3 Dependency on $|S_{11}|$ of the maximum value for envelope correlation in the case of uniform distributed angle of arrival.

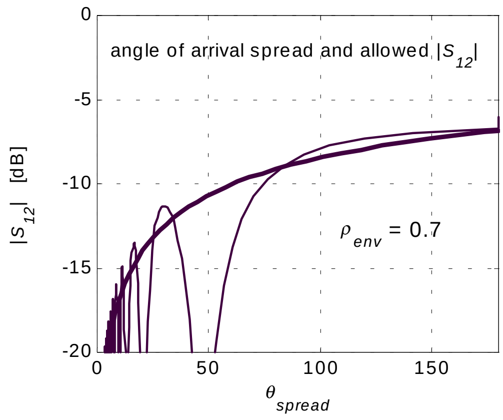

Fig. 4. The allowed level for $|S_{12}|$ depending on spread of arrival angle to keep envelope correlation level $\rho_{env} \leq 0.7$ when $|S_{11}| = 0.5$.

In the case of Fig. 4 the signal with angle spread θ_{spread} is coming from forward direction with the incident angle θ_0 =0°. The oscillating curve depends on the angle of arrival. When θ_0 is 30° and 60° the corresponding shifts of the oscillating curve maximums are about +0.7 dB and –1.5 dB relative to the smooth curve, respectively. According to Fig.3 the envelope correlation depends strongly on $|S_{11}|$ and thus the value $|S_{11}|$ = 0.5 is here only a "practical" value. The position of the curves in Fig. 4 can be adjusted in vertical direction for 180° spread for different $|S_{11}|$ taking in Fig.3 the corresponding $|S_{12}|$ with $\rho_{env}(|S_{12}|)$=0.7.

Conclusions

The presented formulas give an opportunity to control the correlation properties of an antenna array. The scattering parameters contain the whole information of the correlation in the case, when the radiation is coming with uniform angular distribution from several directions. This kind of situation can be found in urban environments, containing different scatterers nearby the mobile. The formulas presented using scattering parameter formalism are convenient to use without simplification also when the mutual coupling is strong. The estimation of the connection between the angular spread and scattering parameter $|S_{12}|$ in the case of envelope correlation 0.7 seems to be realistic and could be placed under practical test.

The uncorrelated noise in array ports is usually an assumption for signal processing algorithms. The noise correlation decreases the system performance. In future systems the role of different interferers and other noise sources is increasing and the model of signals arriving from all directions with different delays can be used as well for the estimation of antenna noise correlation as is done in [5].

References

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